Practice Paper for revision

Applied Mathematics I

(Revised Course)

(03 hours) Total Marks: 80

N.B. (1) **Q.1** is **compulsory.**

(2) Attempt any **three** questions from question **no.2** to question **no.6.**

(3) **Figures** to the right indicates **full** marks.

1. (a) If $x = rcos\theta,\ \ y = rsin\theta$ then find $J\left(\frac{x,y}{r,\theta}\right)$. (03)

(b) Show that $\log(1 + sinx) = x - \frac{x^2}{2} + \frac{x^3}{6} + \cdots$. (03)

(c) If $u = tan^{-1}\left(\frac{y}{x}\right)$ then find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$. (03)

(d) Prove that $\frac{1}{1-\frac{1}{1-\frac{1}{1-cosh^2 x}}} = cosh^2 x$. (03)

(e) Show that every square matrix can be uniquely expressed as the sum of symmetric and skew symmetric matrix. (04)

(f) Find nth Qrder derivative of $y = cosx\ cos2x\ cos3x$. (04)

2. (a) Reduce the matrix into normal form and find its rank (06)

$$A = \begin{bmatrix} 2 & -1 & 1 & 1 \\ 1 & 0 & 1 & 2 \\ 3 & 3 & 3 & 1 \\ 1 & 4 & 2 & 0 \\ 0 & -4 & -1 & 2 \end{bmatrix}.$$

(b) Solve the equation $x^4 - x^3 + x^2 - x + 1 = 0$. (06)

(c) State and prove the Euler's theorem on homogenous function for three variables and hence verify it for $u = \frac{\sqrt{xyz}}{\sqrt{x}+\sqrt{y}+\sqrt{z}}$. (08)

3. (a) Test the consistency of the following system of equations and solve

if consistent.

$$x - 2y + z - t = 2 \; ; \; x + 2y + 2t = 1; \; 4y - z + 3t = -1.$$ (06)

(b) Find stationary values of $x^3 + y^3 - 3axy$. (06)

(c) Separate into real and imaginary part $cos^{-1}(e^{i\theta})$. (08)

4. (a) If $x = uv$, $y = \frac{u}{v}$ prove that $JJ' = 1$.

(b) Prove that $Cos^6\theta + Sin^6\theta = \frac{1}{8}(3Cos4\theta + 5)$. (06)

(c) Solve the following equations by Gauss-Seidal method.

$5x - y = 9; \; -x + 5y - z = 4; \; -y + 5z = 6.$ (08)

5. (a) Show that for real values of a and b, $e^{2ai\,cot^{-1}b}\left(\frac{bi-1}{bi+1}\right)^{-a} = 1$. (06)

(b) If $\lim_{x \to 0} \frac{x(a+bcosx)-csinx}{x^5} = 1$, find the values of a, b, c. (06)

(c) If $y^{1/m} + y^{-1/m} = 2x$ then prove that

$(x^2 - 1)y_{n+2} + x(2n + 1)y_{n+1} + (n^2 - m^2)y_n = 0.$

6. (a) Examine whether the vectors are linearly independent

[3 1 1], [2 0 − 1], [4 2 1]. (06)

(b) If $x^x.y^y.z^z = c$, where $x = y = z = 1$, then show that $\frac{\partial^2 z}{\partial x \partial y} = -(\text{xlogex})^{-1}$.(06)

(c) Fit a straight line for the following data (08)

x:	100	120	140	160	180	200
y:	0.45	0.55	0.60	0.70	0.80	0.85