

(3 Hours)

Total Marks :80

N.B. (1) Question No. 1 is compulsory

(2) Solve any Three from the remaining.

(3) Figure in right indicates full marks.

Q.1(a) Prove that $\frac{1}{1 - \frac{1}{(1 - \sec^2 x)}} = -\sinh^2 x$ 3

(b) If $z(x+y) = (x^2 + y^2)$. Prove that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$ 3

(c) If $u = e^x \cos y, v = e^x \sin y$ find $\frac{\partial(u,v)}{\partial(x,y)}$ 3

(d) Using Maclaurin's series prove that $e^x \log(1+x) = x + \frac{x^2}{2!} + \frac{2x^3}{3!} + \dots$ 3

(e) Prove that the matrix $\frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is Unitary. 4

(f) Find the nth Derivative of $y = \cos x \cos 2x \cos 3x$ 4

Q.2 (a) Find the roots common to $x^4 + 1 = 0$ and $x^6 - i = 0$ 6

(b) Find the nonsingular matrices P and Q such that PAQ is in Normal Form. Also find rank of A 6

Where $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$

(c) State and Prove Eulers theorem for Homogeneous functions on two variables and hence 8

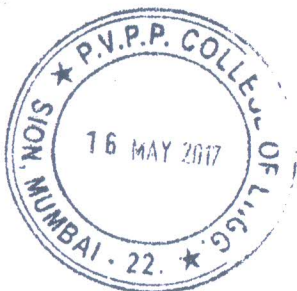
find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ if $u = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log x - \log y}{x^2 + y^2}$

Q.3(a) Determine the values of K for which the following equations are consistent. Also solve 6

the system for these values of k $x + 2y + z = 3, x + y + z = k, 3x + y + 3z = k^2$

(b) Find the stationary values of $f(x,y) = y^2 + 4xy + 3x^2 + x^3$ 6

(c) If $\tan(\alpha + i\beta) = \cos \theta + i \sin \theta$ Prove that $\alpha = \frac{n\pi}{2} + \frac{\pi}{4}$ and $\beta = \frac{1}{2} \log \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$ 8



Turn Over

Q.4(a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ Prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$ 6

(b) Prove that principal value of $(1 + \tan \alpha)^{-i}$ is $e^{\alpha} [\cos(\log \cos \alpha) + i \sin(\log \cos \alpha)]$ 6

(c) Apply Croust method to solve system of linear Equations $x + y + z = 3,$
 $x + 2y + 3z = 4$ 8
 $x + 4y + 9z = 6$

Q.5(a) Find the constants a, b, c so that $\lim_{x \rightarrow 0} \frac{x(a + b \cos x) - c \sin x}{x^3} = 1$ by L'Hospital's rule 6

(b) Prove That $e^{x^2} = e \left[1 + x + x^2 + \frac{5}{6}x^3 + \frac{5}{8}x^4 + \dots \right]$ 6

(c) If $x = \tan(\log y)$ prove that $(1 + x^2)y_{n+1} + (2nx - 1)y_n + n(n-1)y_{n-1} = 0$ 8

Q.6(a) If $A = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & a \\ \frac{2}{3} & \frac{1}{3} & b \\ \frac{2}{3} & -\frac{2}{3} & c \end{bmatrix}$ is orthogonal find a, b, c. 6

(b) Fit a straight line $y = a + bx$ to the following data 6

x	1	2	3	4	5	6
y	49	54	60	73	80	86

(c) If $\frac{x^2}{a^2 + u} + \frac{y^2}{b^2 + u} + \frac{z^2}{c^2 + u} = 1$, prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2 \left(x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} \right)$

Given that u is an implicit function of x, y, z. 8

