

- N.B. : (1) Question No.1 is compulsory.
(2) Attempt any three questions from Question No.2 to Question No.6.
(3) Non-programmable calculator is allowed.

1. (a) Find $L^{-1}\left[\frac{Se^{-\pi s}}{s^2+2s+2}\right]$ 5

(b) State true or false with proper justification "There does not exist an analytic function whose real part is $x^3 - 3x^2y - y^3$ ". 5

(c) Prove that $f_1(x) = 1$, $f_2(x) = x$, $f_3(x) = \frac{(3x^2 - 1)}{2}$ are orthogonal over $(-1, 1)$. 5

(d) Using Green's theorem in the plane, evaluate $\int_C (x^2 - y)dx + (2y^2 + x)dy$ around the boundary of the region defined by $y = x^2$ and $y = 4$. 5

2. (a) Find the fourier cosine integral representation of the function $f(x) = e^{-ax}$, $x > 0$ 6

and hence show that $\int_0^{\infty} \frac{\cos ws}{1+w^2} dw = \frac{\pi}{2} e^{-x}$, $x \geq 0$.

(b) Verify laplaces equation for $U = \left(r + \frac{a^2}{r}\right) \cos\theta$ Also find V and f(z). 6

(c) Solve the following eqn. by using laplace transform. $\frac{dy}{dt} + 2y + \int_0^t y dt = \sin t$ given that $y(0) = 1$. 8



3. (a) Expand $f(x) = \begin{cases} \pi x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$ with period 2 into a fourier series. 6

(b) A vector field is given by $\vec{F} = (x^2 + xy^2)\mathbf{i} + (y^2 + x^2y)\mathbf{j}$ show that \vec{F} is irrotational and find its scalar potential. 6

(c) Find the inverse z - transform of - 8

$$f(z) = \frac{z+2}{z^2-2z+1}, |z| > 1$$

4. (a) Find the constants 'a' and 'b' so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at $(1, -1, 2)$ 6

(b) Given $L(\operatorname{erf} \sqrt{t}) = \frac{1}{S\sqrt{S+1}}$, evaluate $\int_0^{\infty} t e^{-t} \operatorname{erf}(\sqrt{t}) dt$ 6

(c) Obtain the expansion of $f(x) = x(\pi - x)$, $0 < x < \pi$ as a half-range cosine series. 8

Hence show that - (i) $\sum_1^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$

(ii) $\sum_1^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$

5. (a) If the imaginary part of the analytic function $W=f(z)$ is $V = x^2 - y^2 + \frac{x}{x^2 + y^2}$ find the real part U. 6

(b) If $f(k) = 4^k U(K)$ and $g(k) = 5^k U(K)$, then find the z- transform of $f(k) \cdot g(k)$. 6

(c) Use Gauss's Divergence theorem to evaluate $\iint_S \vec{N} \cdot \vec{F} ds$ where $\vec{F} = 4xi + 3yj - 2zk$ and S is the surface bounded by $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$. 8



QP Code : NP-18619

3

6. (a) Obtain complex form of Fourier series for $f(x) = \cos h 3x + \sin h 3x$ in $(-3, 3)$. 6
- (b) Find the inverse Laplace transform of $\frac{(s-1)^2}{(s^2 - 2s + 5)^2}$ 6
- (c) Find the bilinear transformation under which 1, i, -1 from the z-plane are mapped onto 0, 1, ∞ of w-plane. Also show that under this transformation the unit circle in the w-plane is mapped onto a straight line in the z-plane. Write the name of this line. 8



Con. 9833-14.

